MEASURING SPACE-TIME ACCESSIBILITY BENEFITS WITHIN TRANSPORTATION NETWORKS: BASIC THEORY AND COMPUTATIONAL PROCEDURES

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Abstract: Accessibility is a fundamental but often neglected concept in transportation analysis and planning. Three complementary views of accessibility have evolved in the literature. The first is the constraints-oriented approach, best implemented by Hägerstrand's space-time prisms. The second perspective follows a spatial interaction framework and derives "attraction-accessibility measures" that compare destinations' attractiveness with the travel costs required. A third approach measures the benefit provided to individuals by the transportation/land-use system. This paper reconciles the three complementary approaches by deriving space-time accessibility and benefit measures that are consistent with the rigorous Weibull axiomatic framework for accessibility measures. This research also develops computational procedures for calculating these measures within network structures. This provides realistic accessibility measures that reflect the locations, distances and travel velocities allowed by an urban transportation network. Since their computational burdens are reasonable, they can be applied at the urban-scale using a GIS.

1. INTRODUCTION

Accessibility is a fundamental but often neglected concept in transportation analysis and planning. Very roughly, "accessibility" is a measure of an individual's freedom to participate in activities in the environment (Weibull 1980). Despite its primacy to human activities, many transportation analytical methods (e.g., travel demand modeling) and technologies (e.g., Intelligent Transportation Systems) neglect accessibility and instead focus on measuring or maximizing system throughput. While throughput is related, it is peripheral to the true objective of transportation, i.e., maximizing accessibility to opportunities in the environment.

Transportation planning and policy should focus more strongly on individuals' accessibility to the environment. Besides being fundamental, measuring accessibility can be a more sensitive assessment technique than trying to predict actual behavior (Hägerstrand 1975). The spatial distribution of accessibility, particularly *changes* in accessibility, can tell the planner or policy analyst directly who are the "winners" and "losers" in a given scenario. This crucial and central question is often lost in the cloud of results from modeling exercises, traffic simulations and other demand projections.

Sensitive transportation planning requires *rigorous*, *realistic* and *easily computed* measures of individual accessibility. However, many existing accessibility measures do not meet these criteria. First, there is little agreement with respect to what exactly constitutes "accessibility," at least from a formal perspective (see Morris, Dumble and Wigan 1979). Second, accessibility is often measured using very sterile representations of the environment. Often neglected are critical environmental features such as the distances and travel velocities imposed by the urban transportation system. This is very important given the increasingly congested condition of transportation networks.

This research develops rigorous, realistic and easily computed accessibility measures. With respect to rigor, the measures in this paper reconcile three complementary perspectives that have developed in the literature. These perspectives are: i) the *constraints-oriented approach*, best implemented using Hägerstrand's (1970) *space-time prisms*, that measures limitations on individuals' freedom of action in the environment; ii) *attraction-accessibility measures* that use a spatial interaction framework to assess the range of available opportunities with respect to their attractiveness and travel costs, and; iii) *benefit measures* that assesses the "welfare" accruing to individuals. While each perspective has its strengths, each neglects other worthwhile dimensions of accessibility. A constraints-oriented approach treats each opportunity as equal without distinguishing differences among attractiveness and travel costs. Conversely, attraction-accessibility and benefit measures generally do not consider temporal constraints or the time available for activity participation at locations. These space-time contextual effects can be critical for capturing individual variations in accessibility (see Kwan 1998).

This paper uses two principles to tie the three perspectives together. The first principle is an individual-level utility function recently analyzed by Hsu and Hsieh (1997) but also due to Burns (1979). This function considers the attractiveness of each opportunity, the travel time required and the remaining time available for activity participation. This implicitly defines a space-time prism. The second principle is an axiomatic framework developed by Weibull (1976, 1980). This framework provides rigorous guidelines for developing accessibility measures that are consistent internally and with respect to the behavioral situation being analyzed. The accessibility measures derived in this paper meet the requirements of Weibull's system.

With respect to realism, this paper also develops computational procedures for computing the accessibility measures within network structures. This allows evaluation with respect to the locations, distances and travel velocities dictated by the transportation network. Since the procedures have reasonable computational burdens, they can be implemented at the urban-scale using a GIS. Implementing these measures within a GIS allows evaluation using real-world transportation networks, cartographic visualization of model results and linking model results with other georeferenced social, economic and infrastructure data. This will greatly facilitate the use of the measures in real-world transportation planning problems.

Some recent efforts have attempted to incorporate more realistic representations of the spatial environment into accessibility measurement. In particular, Geertman and Van Eck (1995) use a GIS to incorporate network-based travel times into accessibility measurement. While this is an improvement, they use a potential model that does not incorporate temporal constraints nor provide benefit measures. Also, they do not define accessibility within the transportation network; rather, they generate planar space "potential surfaces" that reflect network distances. In contrast, this paper advances GIS-based accessibility measurement by deriving space-time accessibility benefit measures defined *within* the transportation network. Incorporating temporal constraints, benefit measures and the network structure provides more theoretically-consistent and realistic accessibility measures.

This paper focuses on the individual level. This is for three reasons. First, attempting to develop corresponding measures at the aggregate-level raises issues regarding defining and measuring "time budgets" for population aggregates. For example, simply measuring the average time available for individuals in a census tract introduces measurement error and is not satisfactory. Second, while deriving individual-level accessibility measures are straightforward, deriving similar aggregate measures requires more finesse, particularly in the transportation context. Since travel demands are interrelated, we must be careful to capture correctly the demand shifts that can occur when measuring aggregate-level benefits (see Jara-Diaz and Friesz (1982) and some brief comments in the sequel). The third reason is that individual-level measures have value independent of aggregate-level measures, particularly given the ability of GIS to handle large georeferenced datasets. For these reasons this paper concentrates on individual-level accessibility measures; subsequent investigation will address aggregate-level measures and its use in broader location and transportation models

The next section of this paper reviews existing literature in space-time constraints, attraction-accessibility measures and benefit measures. Discussion of attraction-accessibility measures focuses on Weibull's (1976, 1980) axiomatic system rather than accessibility measures per se; for general reviews of accessibility measures, see Dalvi and Martin (1976), Morris, Dumble and Wigan (1979) or Pooler (1987). Section 3 develops the basic theory for measuring space-time accessibility benefits in planar space. Section 4 develops computational procedures for deriving these measures within network structures. Section 5 provides example calculations. Section 6 provides concluding comments.

2. THEORETICAL FRAMEWORK

2.1. Space-time Constraints

Hägerstrand's (1970) space-time framework provides an effective implementation of the constraints-oriented approach to accessibility measurement. The space-time framework provides a powerful and elegant perspective from which to analyze individuals' accessibility to the environment. The space-time framework recognizes that activity participation has both spatial and temporal dimensions, i.e., activities occur at specific locations for finite temporal durations. In addition, the transportation system dictates the velocities at which individuals can travel and therefore the time available for activity participation at dispersed locations. The space-time framework dictates the necessary (but not sufficient) conditions for virtually all human interaction (Burns 1979; Hägerstrand 1970; Pred 1977).

Evidence suggests that temporal constraints can affect the ability of individuals to participate in activities, i.e., accessibility to the environment. Landau, Prashker and Alpern (1982) and Landau, Prashker and Hirsh (1981) found that adding temporal constraints to a spatial choice model improved its predictive ability, albeit by a modest amount. Thill and Horowitz (1997a) develop a two-stage, logit-based choice model that includes temporal constraints. This model had superior fit to empirical data than a non-temporally-constrained model. A simulation analysis by Thill and Horowitz (1997b) found more substantial improvements in predictive accuracy when time constraints are assumed to be probabilistic rather than deterministic.

Several authors have used the space-time framework to assess accessibility directly. The fundamental accessibility construct is the *space-time prism*, that is, the set of locations in space-time that are accessible to an individual given the locations and duration of *fixed* (mandatory) activities, a time "budget" for *flexible* (discretionary) activity participation and the travel velocities allowed by the transportation system (Hägerstrand 1970). An early study by Lenntorp (1976, 1978) simulates all possible activity schedules within an urban environment given space-time prism constraints. The number of possible activity schedules is a measure of accessibility. Burns (1979) assesses changes in the volume of the space-time prism that results from changes in travel velocities, transportation network configuration (planar travel versus restriction to a finegrid network) and changes in mandatory activity scheduling (e.g., flex-time work schedules). Similarly, Hall (1983) examines the impact of travel time randomness (e.g., congestion) and limited information about the environment on the space-time prism.

Several authors measure more directly the *benefits* that result from space-time accessibility. Burns (1979) develops several benefit measures by formulating space-time utility functions. Burns (1979) analytically compares the differences in benefits based on these utility functions and various strategies for improving space-time autonomy. Hsu and Hsieh (1997) use this strategy to examine the benefits from different activity schedule and travel choices, including multistop/multipurpose travel and at-home versus out-of-home activities. However, these are not benefit measures in a strict sense; in both cases the measures simply add the utilities of opportunities in the space-time prism. As noted below, this introduces interpretation problems.

The space-time framework, while powerful, is difficult to operationalize and apply in its classical form as a real-world accessibility measurement tool. A major difficulty is an assumed constant velocity of travel; this is obviously unrealistic in most urban settings. Velocities can change with location, travel direction and time-of-day in urban transportation networks. This is especially true as these networks become increasingly saturated. However, maintaining the data required for detailed computation of urban travel velocities is difficult using traditional data handling methods.

To improve the realism and applicability of the space-time prism approach, Miller (1991) conducts a requirements analysis for computing network-based space-time prism measures using GIS. Miller (1991) shows that these network-based space-time prism measures can be used to evaluate the performance of a transportation system, particularly with respect to spatial variations in accessibility. Kwan and Hong (1998) expand this approach by including cognitive (informational, preferential) constraints within the network-based accessibility calculation. In a comparative analysis of accessibility measures, Kwan (1998) demonstrates that the space-time approach captures activity-based contextual effects neglected by traditional attraction-accessibility measures. This allows more sensitive assessment of individual variations in accessibility, including gender and ethnic differences.

Using the urban transportation network to calculate space-time measures can provide a realistic and practical technique for assessing accessibility. However, Miller (1991), Kwan and Hong (1998) and Kwan (1998) only derive the broad space-time and cognitive constraints on activity participation. Also desirable is a sensitive measure of space-time accessibility benefits. This current research attempts to formulate these measures by reconciling the space-time approach with other theoretical frameworks for measuring accessibility.

2.2. Attraction-accessibility Measures

In a pair of groundbreaking papers, Weibull (1976, 1980) developed an axiomatic framework for formulating attraction accessibility measures (AMs). The axioms stated in Weibull (1976) correspond to properties that AMs should exhibit to obtain internal consistency. These axioms are:

i) Axiom A1. For any configuration of opportunities:

$$f\left[\left\langle (d_1,a_1);\ldots;(d_k,a_k);\ldots;(d_l,a_l);\ldots;(d_n,a_n)\right\rangle\right]=$$
$$f\left[\left\langle (d_1,a_1);\ldots;(d_l,a_l);\ldots;(d_k,a_k);\ldots;(d_n,a_n)\right\rangle\right];$$

ii) Axiom A2.

a) $d_k \le d'_k \Rightarrow f\left[\left\langle (d_k,a_k)\right\rangle_{k=1}^{n}\right] \ge f\left[\left\langle (d'_k,a_k)\right\rangle_{k=1}^{n}\right];$

b) $a_k \le a'_k \Rightarrow f\left[\left\langle (d_k,a_k)\right\rangle_{k=1}^{n}\right] \le f\left[\left\langle (d_k,a'_k)\right\rangle_{k=1}^{n}\right]$

iii) Axiom A3. $f\left[\left\langle (0,a)\right\rangle\right]$ is continuous and increasing;

iv) Axiom A4. $f\left[\left\langle (d_1,a_1);(d_2,a_2)\right\rangle\right] < \lim_{a\to\infty} f\left[\left\langle (0,a)\right\rangle\right] \; \forall \left\langle (d_1,a_1);(d_2,a_2)\right\rangle$

v) Axiom A5.

a) $f\left[\left\langle (d,0)\right\rangle \cup \bar{x}\right] = f[\bar{x}] \; \forall d, \bar{x}$;

b) $f\left[\left\langle (d,0)\right\rangle\right] = 0 \; \forall d$;

vi) Axiom A6. $f[\bar{x}'] = f[\bar{x}''] \Rightarrow f[\bar{x}' \cup \bar{x}] = f[\bar{x}'' \cup \bar{x}] \; \forall \; \bar{x}$

where d_i is the distance to opportunity i, a_i is the opportunity's attraction and $\bar{x} = \left\langle (d_k,a_k)\right\rangle_{k=1}^{n}$ is a configuration of n opportunities.

Axiom A1 states that the order in which opportunities are presented should be irrelevant. Axiom A2 states the value of an AM should be nonincreasing in distance and nondecreasing in activity attraction. Axiom A3 requires an AM evaluated for an opportunity at zero distance to be continuous and increasing. Axiom A4 requires a single opportunity with infinite attraction at zero distance to be better than any pair of opportunities with finite attractions. Axiom A5 states that an opportunity with zero activity attraction should not contribute to the value of an AM. Finally, Axiom A6 is a preferential independence requirement: if two sets of opportunities are equivalently accessible, then adding the same, new opportunity to both sets should not change this equivalence.

Any AM that satisfies axioms A1 - A6 is a *standard attraction-accessibility measure* (SAM). Weibull (1976) proves that any SAM will have the following formal structure:

$$f\left[\left\langle (d_k, a_k)\right\rangle_{k=1}^{n}\right] = g\left(z(d_1, a_1)\oplus\ldots\oplus z(d_n, a_n)\right) \qquad \textbf{(1)}$$

where: $g\colon \Re_+ \to \Re$ is a continuous and increasing function satisfying $g(0) = 0$, z is a *standard distance substitution function* and $\oplus$ is a *standard binary operation.* A standard distance substitution function is any function $z\colon \Re_+^2 \to \Re$ that has the following properties: a) $z(a,0) = z(a)$ (i.e., the substitute attraction of an opportunity at zero distance equals the original attraction); b) $d < d' \Rightarrow z(a,d) \geq z(a,d')\ \forall\ a$ (i.e., the function is non-increasing with respect to distance) and; c) $a < a' \Rightarrow z(a,d) \leq z(a',d)\ \forall\ d$ (i.e., the function is non-decreasing with respect to attractiveness). A standard binary operation is any binary operation that is: i) *commutative*; ii) *monotone*; iii) *has 0 as the algebraic unit* (i.e., $a \oplus 0 = 0 \oplus a = a$), and; iv) *associative.*

Weibull (1980) refines the generic mathematical format by deriving the behavioral conditions under which different SAM formats can be realized from equation (1). Note that equation (1) defines an AM by computing a real number measure for each opportunity and aggregating into an overall summary measure using a (standard) binary operation. Weibull (1980) refers to this general format as a *separable* AM.

Typically, the binary operation used in equation (1) is addition; this defines an *additive* AM:

$$f_1\left[\left\langle (d_k, a_k) \right\rangle_{k=1}^{n}\right] = \sum_{k=1}^{n} z(d_k, a_k) \tag{2}$$

AMs such as Hansen (1959), Wilson (1971), Erlander (1977), Erlander and Stewart (1978) and Geertman and van Eck (1995) follow this format. These AMs generally are spatial interaction, potential-type formulas that assume a positive relationship between the number of opportunities available and a location's accessibility while accounting for attractions and the deterring effect of distance.

Another type of separable AM follows microeconomic theory and assumes utility maximizing choices. A *maxitive* AM has the general format:

$$f_2\left[\left\langle (d_k, a_k) \right\rangle_{k=1}^{n}\right] = \max_{k=1\ldots n}\left\{u(d_k, a_k)\right\} \tag{3}$$

where *u(.)* represents the utility of the opportunity to the individual. Finally, the utility-maximizing approach in (3) can be extended to a random utility-based decision process through the following *transform-additive* AM:

$$f_3\left[\left\langle (d_k, a_k) \right\rangle_{k=1}^{n}\right] = G\left\{\sum_{k=1}^{n} h\left[\bar{u}(d_k, a_k)\right]\right\} \tag{4}$$

where $\bar{u}(d_k, a_k)$ indicates the expected utility associated with opportunity (d_k, a_k) and *G*, *h* are real-valued, continuous and strictly increasing functions. AMs such as Williams (1976) and Williams and Senior (1978) follow this format.

An important contribution of Weibull (1980) is the rigorous specification of conditions under which additive, maxitive and transform-additive AMs are suitable for an accessibility measurement situation. First define the following binary relations:

$\bar{x} \preceq \bar{y}$ ≡ "$\bar{y}$ at least as accessible as $\bar{x}$"

$\bar{x} \prec \bar{y}$ ≡ "$\bar{y}$ is more accessible than $\bar{x}$"

$\bar{x} \sim \bar{y}$ ≡ "$\bar{x}$ and $\bar{y}$ are equivalently accessible"

Weibull (1980) proves that additive and transform-additive AMs are suitable if and only if the set *M* of configurations is *non-negative* ($\varnothing \preceq \bar{x} \;\; \forall \; \bar{x} \in M$, where ∅ is the empty set), *complete* ($\bar{x} \vee \bar{y} \in M \Rightarrow \bar{x}, \bar{y} \in M$, where $\vee$ is the conjunction of the two configurations), *closed under conjunction* ($\bar{x}, \bar{y} \in M \Rightarrow \bar{x} \vee \bar{y} \in M$) and the following axioms are satisfied:

i) Axiom B1. If $\bar{x}, \bar{y} \in M$, then $\bar{x} \preceq \bar{y}$ or $\bar{y} \preceq \bar{x}$ or both;

ii) Axiom B2. If $\bar{x}, \bar{y}, \bar{z} \in M$, $\bar{x} \preceq \bar{y}$, $\bar{y} \preceq \bar{z}$ then $\bar{x} \preceq \bar{z}$ *;*

iii) Axiom B3. If $\bar{x}_1 \vee \bar{y}_1$ and $\bar{x}_2 \vee \bar{y}_2 \in M$, $\bar{x}_1 \sim \bar{x}_2$ and $\bar{y}_1 \sim \bar{y}_2$ then $\bar{x}_1 \vee \bar{y}_1 \sim \bar{x}_2 \vee \bar{y}_2$;

iv) Axiom B4. If $\bar{x}, \bar{y}, \bar{z} \in M$ and $\bar{x} \prec \bar{y}$, then $\bar{x} \vee \bar{z} \prec \bar{y} \vee \bar{z}$;

v) Axiom B5. If $\bar{x}, \bar{y}, \bar{z} \in M$ and $\bar{x} \prec \bar{y}$, then $(n+1)\bar{x} \preceq n\bar{y}$ for some positive integer *n*.

Axiom B1 states that a preference among configurations must be made (even if this preference is a tie). Axiom B2 requires transitive relationships among configurations. Axioms B3 and B4 corresponds to preferential independence requirements of Axiom A6. Finally, Axiom B5 requires the preference ordering among two configurations to be preserved for some finite multiplicative increase in the number of both configurations. This situation does not hold if duplication of configurations does not increase utility (e.g., a configuration offers some unique, unmeasured characteristic that cannot be matched or improved on through duplicating the other configuration). Axioms B1 - B5 imply the following:

vi) Axiom B6. If $\bar{x}, \bar{y} \in M$ and $\varnothing \prec \bar{x} \prec \bar{y}$, then $\bar{y} \preceq n\bar{x}$ for some positive integer *n*.

Since this latter axiom is implied by the first five, it may be easier to check Axiom B6 in a given situation.

Weibull (1980) also proves that a maxitive AM is suitable if and only if the set of configurations M' is countable, complete and Axioms B1, B2 and B7 are satisfied where:

vii) Axiom B7. If $x \vee y \in M'$ and $\bar{x} \preceq \bar{y}$, then $\bar{x} \vee \bar{y} \sim \bar{y}$.

In other words, the conjunction of two configurations is equal to the best of the two configurations, i.e., more is not necessarily better.

Although Weibull's axiomatic system is a powerful technique for achieving internal and external consistency in accessibility measures, it does not admit some realistic spatial choice behaviors. For example, Axiom A1 (ordering of alternatives is irrelevant) does not recognize spatial search behavior by imperfectly informed decision-makers. In this case, the information gathering process implies that the order in which alternatives are consider affects their utility (see Maier 1991; Miller 1993). Similarly, a hierarchical information processing strategy in which decision-makers perceive spatial choices as aggregate "clusters" and then make a choice within each cluster (see Fotheringham 1986) is incongruent with Axiom A6 and Axiom B5 (preferential independence from the contents of configurations). A worthwhile topic for further investigation would be to relax these axioms to accommodate these alternative choice behaviors.

2.3. Transportation Benefits and Accessibility

2.3.1. User Benefits

Although the Weibull (1976, 1980) axiomatic system provides a framework for deriving *consistent* AMs, a problem with many AMs is a lack of a strong behavioral foundation. For example, Williams and Senior (1978) argue that additive AMs such as Hansen (1959) are artifacts of a particular modeling system (the spatial interaction framework). Consequently, it is difficult to interpret their values. For example, researchers often interpret Hansen and related measures as a surrogate for an individual's "potential interaction." However, it is unclear exactly what greater "potential interaction" means beyond a simple ordinal relationship.

Rather than measuring some level of potential interaction, AMs should provide a direct measure of the benefit accruing to individuals, either in utility or monetary terms (Ben-Akiva and Lerman 1979). The basic idea is that "accessibility" is closely associated (if not identical) to the benefits provided by a transportation/land-use system to an individual. Defining accessibility as a measure of user benefits allows access to rigorous theoretical frameworks for interpreting AM values and especially metric changes in these values.

Utility-maximizing choice behavior implies that the benefit received by an individual is the maximum utility of a choice set (Ben-Akiva and Lerman 1979). Limitations to predicting behavior at an individual level often result in treating choice utilities as measurable only up to a stochastic residual. The resulting *random utility framework* choice probabilities are:

$$P_i\left(j|\mathbf{u}\right) = \Pr\left\{u_{ij} - u_{ik} \geq \varepsilon_{ik} - \varepsilon_{ij} \forall j, k \in \mathbf{C_i},\ j \neq k\right\} \quad \textbf{(5)}$$

where u_{ij}, u_{ik} are the measured utilities and $\varepsilon_{ij}, \varepsilon_{ik}$ are the unmeasured utilities for alternatives *j*,*k* and $\mathbf{C_i}$ is choice set for individual *i*. Under this behavioral assumption, a possible accessibility measure is the expected maximum utility of the choice situation:

$$A_i = \mathrm{E}\left[\max_{j \in \mathbf{C}_i} U_{ij}\right] \quad \textbf{(6)}$$

where $U_{ij} = u_{ij} + \varepsilon_{ij}$. The functional form of equation (6) depends on the random utility choice model specified for equation (5); this in turn depends on the assumed distribution of ε_{ij}.

To interpret equation (6) as an accessibility measure, we require two properties (Ben-Akiva and Lerman 1979, 1985). First, expected utility must be nondecreasing with respect to choice set size. This holds for all random choice models in which the expected maximum utility can be defined for each choice set of interest. Second, it must be nondecreasing with respect to the systematic (measurable) utilities. This does not hold for all random choice models. To ensure this second property, we can require the unmeasured utilities in equation (5) to be *translationally invariant*. This means that a change in the systematic utilities translates the joint utility distribution without changing its functional form. Under this restriction, the following conditions hold (Ben-Akiva and Lerman 1979, 1985):

$$\frac{\partial\, \mathrm{E}\left[\max_{j \in \mathbf{C}_i} U_{ij}\right]}{\partial\, u_{ij}} = P_i\left(j|\mathbf{u}\right) \quad \textbf{(7)}$$

$$\sum_{j\in\mathbf{C_i}}\frac{\partial\,\mathrm{E}\left[\max_{j\in\mathbf{C}_i}U_{ij}\right]}{\partial\,u_{ij}}=1 \tag{8}$$

Equation (7) states that the partial derivative of the expected maximum utility with respect to the systematic utility of an opportunity is the choice probability of that opportunity. Equation (8) follows from equation (7) and implies that an increase in the systematic utility of every alternative by a fixed amount increases the accessibility measure by that amount. These conditions satisfy the requirement for the measure to be nondecreasing with respect to systematic utilities.

We can now define an accessibility measure based on the expected maximum utility of a choice situtation (Ben-Akiva and Lerman 1979):

$$A_i=\mathrm{E}\left[\max_{j\in\mathbf{C}_i}u_{ij}\right]=\sum_{j\in\mathbf{C_i}}\int P\left(j|\mathbf{u}\right)d\mathbf{u} \tag{9}$$

A convenient, translationally invariant specification for the unobserved utilities is the Type I extreme value distribution. In this case, $P\left(j|\mathbf{u}\right)$ can be calculated using the familiar and tractable logit model and equation (9) reduces to:

$$A_i=\frac{1}{\mu}\ln\sum_{j\in\mathbf{C}_i}\exp\left(\mu u_{ij}\right)+K \tag{10}$$

where μ is the scale parameter for the extreme value distribution and K is the constant of integration. However, since we would like the accessibility measure to be zero when all systematic utilities are zero, it is reasonable to set $K=0$. If we scale the utilities so that μ is equal to unity, equation (10) becomes:

$$A_i=\ln\sum_{j\in\mathbf{C}_i}\exp\left(u_{ij}\right) \tag{11}$$

i.e., the common expression for the expected maximum utility given a logit choice mechanism. As Ben-Akiva and Lerman (1985) and Small (1992) note, we can also express this measure in travel cost units through dividing equation (11) by a travel cost coefficient.

Since metric differences in expected maximum utility are meaningful, we can also calculate the change in accessibility for individual *i* given a change in the observable utilities (Ben-Akiva and Lerman 1979, 1985):

$$\Delta A_i = \sum_{j \in \mathbf{C_i}} \int_{\mathbf{u_i^1}}^{\mathbf{u_i^2}} P(j|\mathbf{u}) d\mathbf{u} \tag{12}$$

where $\mathbf{u_i^1}, \mathbf{u_i^2}$ are the initial and new measured utilities (respectively). If we assume the Type I extreme value distribution equation (12) reduces to:

$$\Delta A_i = \frac{1}{\mu} \ln \sum_{j \in \mathbf{C}_i^2} \exp(\mu u_{ij}^2) - \frac{1}{\mu} \ln \sum_{j \in \mathbf{C}_i^1} \exp(\mu u_{ij}^1) \tag{13}$$

Equation (13) is the difference in expected maximum utilities between the two choice situations.

Several authors (e.g., Ben-Akiva and Lerman 1979, 1985; Jara-Diaz and Friez 1982; Jara-Diaz and Farah 1988; Small 1992; Small and Rosen 1981; Williams 1976, 1977; Williams and Senior 1978) note a correspondence between expected maximum utility and a classic benefit measure in microeconomic theory, namely *consumer surplus*. Consumer surplus measures the benefit accruing to individuals as a function of the prevailing market price or price changes. In a multiple good market with multiple price changes (e.g., a transportation market), calculating consumer surplus requires evaluating a line integral between the previous and new cost vectors (Glaister 1974; Hotelling 1938). This line integral is unambiguous only if the demand cross-elasticities are equal; this corresponds to Green's theorem regarding the uniqueness of line integrals. A corresponding property within the random utility framework follows from the requirement of unobserved utilities to be translationally invariant (Ben-Akiva and Lerman 1985):

$$\frac{\partial P_i(j|\mathbf{u})}{\partial u_{ik}} = \frac{\partial P_i(k|\mathbf{u})}{\partial u_{ij}} \tag{14}$$

As demonstrated by Williams (1976, 1977) and Williams and Senior (1978), this condition allows calculation of consumer surplus using expected maximum utility.

Jara-Diaz and Farah (1988) review consumer surplus measures for transportation markets. They note that the "log-sum" expected maximum utility measures that follow from the Type I extreme value residual assumption support the two major consumer surplus calculations. For example, *Marshallian consumer surplus* (that is, consumer's willingness-to-pay above the prevailing market price) is derived by defining μ as the scale parameter of the extreme value distribution (as in equations (10), (11) and (13)). Although this measure is in utility terms, we can express it a cost metric by dividing the measure by a travel cost coefficient. Conversely, we can define the parameter as the marginal utility of income (i.e., $\mu \equiv \partial u_{ij} / \partial y_i$ where y_i is the individual's income; see Small (1992)). In this case, we derive the Hickesian *compensating variation* for market cost changes from the log-sum measure. This is the income transfer required to maintain the same utility level using the new costs as a base (see Varian 1992).

Although we can interpret the expected maximum utility measure as a measure of consumer surplus, we should note some cautions. First, it is unclear that consumer surplus measured in this manner is sufficient as a measure of *willingness-to-pay* (that is, a measure that can be interpreted unambiguously in monetary terms) unless certain utility function conditions hold. McFadden (1995, 1998) argues that the log-sum measure cannot be interpreted as willingness-to-pay unless the utility function is linear with respect to income.

A related caveat concerns deriving consumer surplus measures at the aggregate (i.e., demand function) level. Some authors (most notably, Cochrane (1975) and Neuberger (1971)) have invoked analogous cross-elasticities condition for travel demand situations and derived consumer surplus measures from spatial interaction and other related travel demand models. These are equivalent to log-sum measures since multinomial logit and spatial interaction models are equivalent formally (Anas 1983). However, the cross-elasticity assumption is less innocent at the aggregate level. Jara-Diaz and Friesz (1982) demonstrate that equivalent cross-elasticities are not consistent with the theoretically-correct directions for demand shifts in transportation markets. This does not prevent deriving consumer surplus measures for transportation markets since these only require a theoretically-consistent *single* path for the line integral. In contrast, equivalent cross-elasticities imposes the stronger condition that *all* paths are equal

Although a willingness-to-pay measure is valuable, our present concern is developing a practical measure that reflects benefits and therefore can be used to assess metric differences in accessibility. Consequently, we justify and interpret the log-sum benefit measure in utility rather than consumer surplus terms. Obtaining a strict consumer surplus measure introduces difficult data and measurement issues that may diminish the pragmatic nature of these accessibility measures. Nevertheless, we note the correspondence with consumer surplus since developing rigorous linkages is a worthwhile topic for theoretical investigation (and perhaps eventual development of empirical space-time willingness-to-pay measures).

2.3.2. Locational Benefits

Wilson (1976) provides another approach to measuring benefits obtained through spatial interaction between specified locations. Although derived at the travel demand-level, we can apply the basic concept at the individual-level. Wilson (1976) notes that given a spatial interaction model such as the following production-constrained version:

$$S_{ij} = \frac{O_i a_j^{\alpha} \exp(-\lambda c_{ij})}{\sum_k a_k^{\alpha} \exp(-\lambda c_{ik})} \qquad \textbf{(15)}$$

where O_i is the number of people in origin i, a_j is the attractiveness of destination j, and c_{ij} is the interaction cost, we can write $a_j^{\alpha} = \exp(\alpha \ln a_j)$ and then rewrite equation (15) as:

$$S_{ij} = \frac{O_i \exp\left[\lambda\left(\frac{\alpha}{\lambda}\ln a_j - c_{ij}\right)\right]}{\sum_k \exp\left[\lambda\left(\frac{\alpha}{\lambda}\ln a_j - c_{ik}\right)\right]} \qquad \textbf{(16)}$$

Given this format, $\frac{\alpha}{\lambda}\ln a_j - c_{ij}$ can be interpreted as the net interaction benefits (benefits minus cost) for an individual at i who chooses location j; the consumer's decision problem is to maximize these net benefits. We can measure the total welfare level of consumers as:

$$CW = \sum_i \sum_j S_{ij}\left(\frac{\alpha}{\lambda}\ln a_j - c_{ij}\right) \qquad \textbf{(17)}$$

Coehlo and Wilson (1976) show that maximizing the consumer welfare measure (17) is equivalent to maximizing (Marshallian) consumer surplus.

Beckmann, Golob and Zahavi (1983a) develop a related approach to measuring accessibility benefits that uses the space-time prism directly. Similar to Wilson (1976), they measure the benefits derived from patronizing a given activity location as the difference between the utility of that location and the required cost, in this case measured by travel time. After making some simplifying spatial assumptions (constant velocity of travel, monocentric urban form with smooth density decay from the urban center), they derive a geometric expression for the accessibility benefits at a given location in the urban area. This geometric construct is the intersection between the space-time prism and the urban opportunity density field. An alternative derivation using random utility theory allows definition of an urban travel probability field. A subsequent paper finds good correspondence between these probability fields and travel characteristics in Wahington, D.C. (Beckmann, Golob and Zahavi 1983b).

3. SPACE-TIME ACCESSIBILITY BENEFITS: BASIC THEORY

This section develops basic space-time SAMs that reconcile the space-time constraint, attraction-accessibility and consumer surplus approaches. The point of departure is a space-time activity utility function analyzed by Hsu and Hsieh (1997) and Burns (1979). Discussed first is the general utility function and its properties relative to Weibull's (1976) axioms. Developed next is a specific utility function relevant to the choice situation considered in this paper. Planar space additive and transform-additive SAMs follow next. These measures are extended to the transportation network case in the sequel.

3.1. <u>Utility Functions</u>

3.1.1. Generic Utility Function

Consider the following generic utility function (Burns 1979; Hsu and Hsieh 1997):

$$u(a,T,t) = a^{\alpha} T^{\beta} \exp(-\lambda\, t) \tag{18}$$

where:

a = attraction of activity location

T = time available for activity participation; $T = f(t)$

$t =$ travel time required

$a, t, T, \alpha, \beta, \lambda \geq 0$

This utility function postulates a non-compensatory decision-making process involving the attraction of the activity location, the time available for activity participation and the travel time required. Note that the utility function implicitly defines a space-time prism since utility is zero if activity participation time is zero or negative.

We now confirm that the space-time utility function is a standard distance substitution function in the Weibull (1976)-sense. Recall the properties of a standard distance substitution function (section 2.2). We now note the following properties of equation (18):

$$u(a, T, 0) = a^{\alpha} T^{\beta} \exp(-\lambda\, 0) = a^{\alpha} T^{\beta} \qquad \textbf{(19)}$$

$$\frac{\partial u}{\partial t} = -\lambda a^{\alpha} T^{\beta} \exp(-\lambda t) \qquad \textbf{(20)}$$

$$\frac{\partial u}{\partial a} = \alpha\, a^{(\alpha-1)} T^{\beta} \exp(-\lambda t) \qquad \textbf{(21)}$$

$$\frac{\partial u}{\partial T} = \beta T^{(\beta-1)} a^{\alpha} \exp(-\lambda t) \qquad \textbf{(22)}$$

Equation (19) satisfies the requirement that the substitute attraction at zero distance equals the original attraction. The partial derivative (20) is negative or zero over the required domain, satisfying the condition that the function is non-increasing with respect to travel time. The partial derivatives (21) and (22) are positive or zero over the required domains, satisfying the condition that the function is non-decreasing with respect to attractiveness measures. Therefore, equation (18) is a standard distance substitution function.

Since we are interested in developing transform-additive, additive and maxitive SAMs in the sequel, we must determine the decision contexts that are consistent with the appropriate Weibull (1980) behavioral axioms. Recall that transform-additive and additive SAMs are appropriate if axiom B6 holds; this implies that several opportunities with low net attractiveness can provide the same or greater accessibility as one opportunity with high net attractiveness. A maxitive SAM is appropriate for situations where make transitive preferences (B1 and B2) and only consider the best element of any choice set (B7). As discussed previously, these SAMs also introduce some requirements on the choice sets; however, in most empirical settings we can assume these requirements are met as long as all relevant opportunities are included (i.e., there is no arbitrary elimination of opportunities; see Weibull (1980)).

Weibull (1980) notes that axiom B6 holds as long as no opportunity offers unique features that cannot be compensated by other opportunities. Imperfectly informed decision-makers could evaluate an opportunity differently depending on the order in which it is considered; this creates uniqueness that may violate axiom B6. Proximity to other opportunities can also create uniqueness that violates B6; proximity may be considered by hierarchical decision-makers or in multi-stop travel situations. Similarly, considering proximity can violate axiom B7 since this creates preferences for opportunities in addition to the best member of the choice set. Therefore, the SAMs developed below will apply to perfectly informed, non-hierarchical decision-makers conducting single-stop/single purpose travel. Despite its limitations, this is a commonly analyzed choice situation. As stated previously, a worthwhile extension of this research is to relax these axioms to create AMs that encompass wider ranges of behaviors.

3.1.2. Specific Utility Function

We can now specify the generic utility function in equation (18). Assume that an individual cannot leave a fixed activity at location *i* until t_i and must participate in another fixed activity at location *j* by t_j (*i* and *j* may be the same location). First define the utility of participating in an activity at location *k* as:

$$u_{ij}\left(a_k, T_k, t_k\right) = a_k^{\alpha} T_k^{\beta} \exp\left(-\lambda t_k\right) \qquad \textbf{(23)}$$

where:

$$t_k = \left(d\left(\mathbf{x}_i, \mathbf{x}_k\right) + d\left(\mathbf{x}_k, \mathbf{x}_j\right)\right) v^{-1} \qquad \textbf{(24)}$$

$\mathbf{x}_i =$ location vector for *i*

$d\left(\mathbf{x}_i, \mathbf{x}_k\right) =$ distance from location *i* to location *k*

$v =$ constant velocity of travel

$$T_k = \begin{cases} t_j - t_i - t_k > 0 \\ 0 \ \text{else} \end{cases} \quad \textbf{(25)}$$

Note that equation (25) does not include a minimum threshold for activity participation time, i.e., any positive participation time has utility (although this utility can be low depending on the value of the parameter β). For generality a minimum threshold time will be left unspecified, although this and other measures (such as delay time) could be easily accommodated (see Kwan and Hong 1998). If data on the mandatory end time (t_i) and start time (t_j) for the fixed activities are not available, we can replace these variables with a simple, exogenous travel "time budget" T' (e.g., reported by the individual). These latter data may be more readily available.

3.2. <u>Standard Attraction-accessibility Measures</u>

Based on the location-specific utility functions, we can now define additive and transform-additive SAMs. These measure the benefit of the space-time prism defined by fixed activity locations *i* and *j*, time budget (t_j - t_i) or T' and a constant velocity of travel *v*. A transform-additive SAM based on a logit decision process is:

$$AM_1 = \frac{1}{\lambda} \ln \sum_{k=1}^{m} \exp\left(a_k^{\alpha} T_k^{\beta} \exp(-\lambda t_k)\right) \quad \textbf{(26)}$$

This is the expected maximum utility of the opportunities within the space-time prism. Since this AM uses the space-time utility function directly, we only need to confirm that addition is a standard binary operation and ln(.) and exp(.) are real-valued, continuous and increasing functions. Clearly, this is the case; therefore, AM_1 being a SAM follows from equation (18) being a standard distance substitution function.[1]

An additive SAM provides the total locational benefits in the space-time prism. Using the Wilson (1976) transformation, first define the following locational benefit measure based on the space-time utility function:

$$b_k = \begin{cases} 0 & \text{if } a_k = 0 \text{ or } T_k \le 0 \\ \exp\left[\lambda\left(\frac{\alpha}{\lambda}\ln a_k + \frac{\beta}{\lambda}\ln T_k - t_k\right)\right] & \text{else} \end{cases} \quad \textbf{(27)}$$

Note that, in contrast to equation (17), equation (27) retains the natural exponent function in its second component; this ensures a non-negative benefit measure. Since this is a one-to-one transformation, it retains the key properties of the Wilson (1976) consumer welfare measure. Based on this definition, an additive AM is:

$$AM_2 = \sum_{k=1}^{m} b_k \tag{28}$$

where m is the number of flexible activities.

Equation (28) involves a standard binary operation (addition); therefore, to establish that AM_2 is a SAM requires confirming that equation (27) retains the properties of a standard distance substitution function. First, we can note that the partial derivatives calculated for equation (18) are also the corresponding partials for equation (27), establishing that equation (27) is non-increasing with respect to travel time and non-decreasing with respect to attractiveness measures over the required domains. The remaining task is showing that $b(a,T,0)$ returns the original attractiveness. Equations (29) and (30) establish this for the cases a, $T>0$ and $a = 0$, $T = 0$ (respectively):

$$\begin{aligned} b(a,T,0) &= \exp\left[\lambda\left(\frac{\alpha}{\lambda}\ln a + \frac{\beta}{\lambda}\ln T\right)\right] = \exp(\alpha \ln a + \beta \ln T) \\ &= a^{\alpha} T^{\beta} \quad \forall\, a,T > 0 \end{aligned} \tag{29}$$

$$b(0,T,0) = b(a,0,0) \equiv 0 = (0)T^{\beta} = a^{\alpha}(0) \tag{30}$$

Therefore, equation (27) retains the properties of a standard distance substitution function and AM_2 is a SAM.

For completeness, we can also form a maxitive AM. Recall that this type of AM assumes utility maximizing choices in the traditional microeconomic-sense (i.e., the utility function is completely measurable). Since equation (27) is a one-to-one transformation of equation (23), a utility maximizing choice also maximizes equation (27). We can measure the individual's accessibility benefit as the maximum locational benefit in the choice set:

$$AM_3 = \max_{\{k\}}[b_k] \tag{31}$$

Since max[.] is a standard binary operation, AM_3 is a SAM.

4. SPACE-TIME ACCESSIBILITY WITHIN TRANSPORTATION NETWORKS

This section extends the planar space AMs developed previously to the more realistic setting of a transportation network. This allows the AMs to reflect the distances, travel velocities and locations dictated by an urban transport network. Note that, in contrast to Geertman and Van Eck (1995), we will define the AMs *within* the network rather than simply use network distances in otherwise planar space AMs.

Defining the AMs within the transportation network offers theoretic and pragmatic advantages. From a theoretical perspective, this treatment of the travel environment is more consistent than assuming travel both within and external to the network. From a pragmatic perspective, this will allow us to characterize the transportation network itself with respect to space-time accessibility. The characterized network can be visualized and related to address-indexed data using a GIS.

4.1. Network Locational and Travel Characteristics

We assume the following with respect to activity locations within the network. The flexible activity sites (k) are located at nodes in the network. The fixed activity locations (i and j) are distributed throughout the network arcs. We also restrict our attention to undirected networks at present. Both restrictions allow access to efficient computational procedures without too much damage to the realism of our representation.

Distances and velocities are based on network routes. For simplicity, we will assume shortest path routes between any two locations. The travel time of any route is an additive function of the distances and velocities of each link that comprise that route. Since the fixed activity locations are distributed within network arcs rather than aggregrated at nodes, this measurement must account for partial travel within a given link. Thus, the travel time between two locations $\mathbf{x}_i$ and $\mathbf{x}_j$ is:

$$t(\mathbf{x}_i, \mathbf{x}_j) = \sum_{a_{se} \in r_{ij}} \theta_{se} l_{se} v_{se}^{-1} \tag{32}$$

where a_{se} is an arc between nodes s and e, r_{ij} is the shortest path between locations i and j, $\theta_{se} \in [0,1]$ is the proportion traveled of arc a_{se}, l_{se} is the length of arc a_{se} and v_{se} is the travel velocity within arc a_{se}. Note that, given our locational assumptions, θ_{se} will generally be equal to unity except for the arcs where the fixed activities are located.

4.2. Computational Procedures

4.2.1. Network Transformation Method

The computational procedures used to compute the AMs within a transportation network are based on the network transformation method described in Okabe and Kitamura (1996) and Okabe and Okunuki (1997). This method consists of two stages. In the first stage, we generate an *extended shortest path tree* for each network node of interest (in our case, the activity locations). The shortest path trees are "extended" in the sense that it determines the boundary of the shortest path tree at the sub-arc level. Instead of ending at existing network nodes, the tree is extended to *breakpoint nodes* inserted at locations within an arc at the boundary where two shortest routes to a location meet (see Okabe and Kitamura (1996), p. 332, Fig 1). Inserting breakpoint nodes for all activity locations and generating the new topology creates a network in which all locations within an arc use the same routes to activity locations.

The second phase method builds on the extended shortest path tree by projecting the network into an *accessibility space.* We assign to each node an *m*-dimensional vector showing the shortest path distances (or a function of each distance) to each activity location, where *m* is the number of flexible activities. Combined with the new network topology, this generates a network with the valuable property that a distance-based function defined within each link can be characterized as a linear combination of the distances evaluated at the link's nodes, i.e.:

$$g[\mathbf{y}(\rho_{se})] = g[(1-\rho_{se})\mathbf{y}_s + \rho_{se}\mathbf{y}_e] \qquad \textbf{(33)}$$

where $\mathbf{y}_s = f(\mathbf{x}_s)$ is the location vector for node *s* in accessibility space A^m, $\mathbf{x}_s$ is the location of the node in $\Re^2$, $f\colon \Re^2 \rightarrow A^m$ is the network transformation function and $\rho_{de} \in [0,1]$ characterizes locations within the arc. Equation (33) interpolates the distance-based function for locations within an arc based on the "known" distances at the arc's endpoints.

4.2.2. Computing Network-based AMs

Network transformation. We solve the extended shortest path tree problem from each flexible activity location *k* using link travel times. These travel times can be a function of network flow. Then we insert the breakpoints, generate the new network topology and store the appropriate vector of travel times to flexible activity locations at each node in the extended network.

The shortest path trees rooted at each activity flexible location are sufficient for obtaining the breakpoints and the travel time calculations required for the AMs. Since transportation networks tend to be sparse (i.e., each node is connected to relatively few other nodes), each shortest path tree can be calculated in $O((n_N + n_A)\log n_A)$ steps, where n_A is the number of arcs and n_N is the number of nodes in the network (see Sedgewick 1992). Therefore, the total time complexity is $O(m(n_N + n_A)\log n_A)$, where m is the number of flexible activity locations. Storage requirements are $O(mn_{N^*})$, where n_{N^*} is the number of nodes in the extended network (including breakpoint nodes) (Okabe and Kitamura 1996).

Computing AMs using the transformed network. Once we have computed the extended shortest path trees, generated the extended network topology and assigned the travel time vectors to each node we can calculate the network-based AMs. We can do this in one of three ways. First, we can calculate the accessibility benefits for a given individual. Second, we can interpolate the accessibility benefits distributed within an arc by holding either the first or second fixed activity location static and calculating the accessibility benefits corresponding to varying locations for the other fixed activity location. Finally, we can determine the (fixed activity) locations within the network that exhibit a specified benefit level. We assume that a "start" node and an "end" node characterize each arc in the extended network.

To calculate the accessibility benefits for a given individual (e.g., a member of our sample data), first define the following transformed distance function:

$$t(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j) = (1-\theta_{se}^i)\, t(\mathbf{x}_s^i, \mathbf{x}_k) + \theta_{se}^i t(\mathbf{x}_e^i, \mathbf{x}_k) + (1-\theta_{se}^j)\, t(\mathbf{x}_k, \mathbf{x}_s^j) + \theta_{se}^j t(\mathbf{x}_k, \mathbf{x}_e^j) \qquad \textbf{(34)}$$

where:

$t_k(\mathbf{x}_s^i, \mathbf{x}_k) =$ travel time from the start node for the arc containing fixed activity location i to flexible activity location k; these are computed during the network transformation phase.

$t_k(\mathbf{x}_e^i, \mathbf{x}_k) =$ travel time from the end node for the arc containing fixed activity location i to flexible activity location k.

$\theta_{se}^i \in [0,1] =$ location of fixed activity site i within arc a_{se}^i (relative to start node).

$t_k(\mathbf{x}_k, \mathbf{x}_s^j) =$ travel time to the start node for the arc containing fixed activity location j from flexible activity location k.

$t_k(\mathbf{x}_k, \mathbf{x}_e^j) =$ travel time to the end node for the arc containing fixed activity location j from flexible activity location k.

$\theta_{se}^i \in [0,1] =$ location of fixed activity site j within arc a_{se}^j (relative to start node).

We can now calculate the benefit level associated with this case by using this transformed distance function within the previously defined AMs:

$$AM_1(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j) = \frac{1}{\lambda} \ln\left(\sum_{k=1}^{m} \exp\left(a_k^{\alpha} \left(t_j - t_i - t(\mathbf{x_i}, \mathbf{x}_k, \mathbf{x}_j)\right)^{\beta} \exp\left(-\lambda t_k(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j)\right)\right)\right) \quad \textbf{(35)}$$

$$AM_2(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j) = \sum_{\substack{k=1 \\ a_k>0 \\ T_k>0}}^{m} \exp\left[\lambda\left(\frac{\alpha}{\lambda}\ln a_k + \frac{\beta}{\lambda}\ln\left(t_j - t_i - t_k(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j)\right) - t_k(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j)\right)\right] \quad \textbf{(36)}$$

$$AM_3(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j) = \max_{\{k|a_k>0, T_k>0\}}\left[0, \exp\left[\lambda\left(\frac{\alpha}{\lambda}\ln a_k + \frac{\beta}{\lambda}\ln\left(t_j - t_i - t_k(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j)\right) - t_k(\mathbf{x}_i, \mathbf{x}_k, \mathbf{x}_j)\right)\right]\right] \quad \textbf{(37)}$$

Note that we can use the individual's reported time budget T' in both equations rather than the fixed activity end and start times t_i, t_j if these latter data are not available.

Computing equations (35) and (36) requires m operations. If there are n_I individuals to be evaluated, each equation will require $n_I m$ total operations (the average and worse cases are equivalent). Equation (37) requires computing m values and then finding the maximum of the set. We can find a set's maximum value (equivalently, the minimum value) on average in linear time with respect to its cardinality (Aho, Hopcraft and Ullman 1974, Theorem 3.11). Therefore, the total number of operations required is $n_I(m+m)$. Since this is proportional to $n_I m$, the expected time complexity of all three equations is $O(n_I m)$. This is also the worse case complexity for equations (35) and (36).

The benefit levels calculated in equations (35) - (37) can be "assigned" to any of the activity locations depending on the importance of these locations in the analysis. For example, the first, second or both fixed activity locations may correspond to the individual's home depending on the type of travel being analyzed. We may wish to also maintain referencing to other, non-home activity locations. For example, we may want to compute the benefit levels of employees for flexible activity participation during commute periods. Another possibility is calculating the total benefits for all individuals who may patronize a given flexible activity location such as a public facility. In any event, it seems sensible to maintain pointers from the three activity location types to the benefit level datum. This will allow easy querying and cartographic visualization of the benefit level from multiple perspectives.

In the second case, we wish to "interpolate" the accessibility benefit levels within the network. In this case, we take one of the fixed activity sites as given and interpolate the accessibility benefits that would accrue to varying locations of the other fixed activity sites *given the same travel time restrictions* (t_i and t_j) *or time budget* (T'). For example, these varying locations may be individuals' residences in a study area. Assume that we wish to interpolate benefit levels for varying locations of the first fixed activity location (i.e., $\mathbf{x}_i$ is variable while $\mathbf{x}_j$ remains fixed). In this case, define the following distance function:

$$t\left(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j\right) = (1-\rho_{se}^{i})\, t\left(\mathbf{x}_s^{i}, \mathbf{x}_{\mathbf{k}}\right) + \rho_{se}^{i} t\left(\mathbf{x}_e^{i}, \mathbf{x}_k\right) + (1-\theta_{se}^{j}) t\left(\mathbf{x}_k, \mathbf{x}_s^{j}\right) + \theta_{se}^{j} t\left(\mathbf{x}_k, \mathbf{x}_e^{j}\right) \qquad \textbf{(38)}$$

where:

$\rho_{se}^{i} \in [0,1] =$ variable location of activity site i within arc a_{se} (relative to start node).

Note that in equation (38) θ_{se}^{j} is fixed while ρ_{se}^{i} is variable. We can now "interpolate" the benefit levels within any arc by evaluating $AM_1(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$, $AM_2(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$ or $AM_3(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$ for varying values of ρ_{se}^{i}. The symmetric case ($\mathbf{x}_j$ is variable while $\mathbf{x}_i$ remains static) is straightforward.

$AM_1(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$ and $AM_2(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$ require $n_a n_\rho m$ evaluations, where n_a is the number of arcs to be evaluated n_ρ is the number of interpolated locations within each arc. $AM_3(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$ requires $n_a n_\rho (m+m)$ operations; this is proportional to $n_a n_\rho m$. Also, since n_ρ is likely to be small relative to n_a and m, we can treat n_ρ as a constant in the complexity analysis. Therefore, the expected time complexity for the three AMs is $O(n_a m)$; this is also the worse case complexity for AM_1 and AM_2,.

The third, attribute-based AM calculation requires *solving* equation $AM_1(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$, $AM_2(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$ or $AM_3(\rho_{se}^{i}, \mathbf{x}_k, \mathbf{x}_j)$ for the ρ_{se}^{i} that makes $AM_1 = c_1$, $AM_2 = c_2$ or $AM_3 = c_3$ (respectively) where c_1, c_2 and c_3 are the specified benefit levels to be "mapped" within the network. (Again, the symmetric case is straightforward.) The first step is to identify the arcs that are candidates for containing the given benefit level. For each arc a_{se}, evaluate the following function (depending on the AM) at $\mathbf{x}_s$ and $\mathbf{x}_e$:

$$I(\mathbf{x}, \mathbf{x}_k, \mathbf{x}_j) = AM(\mathbf{x}, \mathbf{x}_k, \mathbf{x}_j) - c \qquad \textbf{(39)}$$

We can then determine the existence of an $AM = c$ location within a_{se} using the following test:

i) If $I(\mathbf{x}_s, \mathbf{x}_k, \mathbf{x}_j) I(\mathbf{x}_e, \mathbf{x}_k, \mathbf{x}_j) < 0$, then c is within arc a_{se}.;

ii) If $I(\mathbf{x}_s,\mathbf{x}_k,\mathbf{x}_j)I(\mathbf{x}_e,\mathbf{x}_k,\mathbf{x}_j)=0$ then c is at node s $(\text{if } I(\mathbf{x}_s,\mathbf{x}_k,\mathbf{x}_j)=0)$, node $e(\text{if } I(\mathbf{x}_e,\mathbf{x}_k,\mathbf{x}_j)=0)$ or is constant throughout arc a_{se} $(\text{if } I(\mathbf{x}_s,\mathbf{x}_k,\mathbf{x}_j)=0 \text{ and } I(\mathbf{x}_e,\mathbf{x}_k,\mathbf{x}_j)=0)$;

iii) If $I(\mathbf{x}_s,\mathbf{x}_k,\mathbf{x}_j)I(\mathbf{x}_e,\mathbf{x}_k,\mathbf{x}_j)>0$ then c is not within arc a_{se} nor at its nodes;

Computing $I_1(\mathbf{x},\mathbf{x}_k,\mathbf{x}_j)$ and $I_2(\mathbf{x},\mathbf{x}_k,\mathbf{x}_j)$ requires $2n_A m$ operations (both average and worse cases) since we must evaluate this measure at both end nodes of each arc. This is proportional to $n_A m$. Computing $I_3(\mathbf{x},\mathbf{x}_k,\mathbf{x}_j)$ requires on average $2n_A(m+m)$ operations due to the max-value search. This is also proportional to $n_A m$. Therefore the expected time complexity for all three measures is $O(n_A m)$; this is also the worse case complexity for $I_1(\mathbf{x},\mathbf{x}_k,\mathbf{x}_j)$ and $I_2(\mathbf{x},\mathbf{x}_k,\mathbf{x}_j)$.

The second step is finding the target location within a candidate arc. Analytical evaluation of $AM_1(\rho^i_{se},\mathbf{x}_k,\mathbf{x}_j)=c_1$, $AM_2(\rho^i_{se},\mathbf{x}_k,\mathbf{x}_j)=c_2$ or $AM_3(\rho^i_{se},\mathbf{x}_k,\mathbf{x}_j)=c_3$ with respect to ρ^i_{se} is difficult. Instead, we can perform a one-dimensional numerical search for the target values using a technique such as the bisection method. Also note that an $AM=c$ location can occur twice within an arc, specifically, with respect to each direction of travel. Therefore, we must represent the undirected arc as two directed arcs oriented in opposite directions and conduct the numeric search independently for both arcs.

The bisection method can find a satisfactory interval in approximately $n_f=\log_2\left(\frac{AM(\mathbf{x}_s,\mathbf{x}_k,\mathbf{x}_j)-AM(\mathbf{x}_e,\mathbf{x}_k,\mathbf{x}_j)}{\delta}\right)$ evaluations of $AM(\rho^i_{se},\mathbf{x}_k,\mathbf{x}_j)$, where δ is the desired accuracy level measured by the size of the interval (see Gill, Murray and Wright 1981). Therefore, the numeric search for $AM=c$ locations across the network can require $2n_A n_f m$ operations AM_1, AM_2 and $4n_A n_f m$ for AM_3. However, in practice this is likely to be much smaller since only a small subset of the network arcs will contain these locations (i.e., $n_a << n_A$, where n_a is the number of candidate arcs from step 1). Therefore, we can conclude that the expected time complexity for all three searches is

$O(n_f m)$ although the worse case is $O(n_A n_f m)$. However, the worse-case is very unlikely: this occurs when *all* network arcs are candidates.

5. EXAMPLES

We have developed a prototype GIS-AM software system involving three interfaced components: i) ARC/INFO® GIS software; ii) a C++ based AM toolkit; and, iii) an Arc Macro Language®-based graphical user interface. This section provides some example AM calculations generated from this system.

Figure 1 provides the input network. Network nodes correspond to the "original" nodes as well as the inserted breakpoint nodes. Arc labels correspond to arc travel times in minutes. A "start" fixed activity location (*i*) and an "end" fixed activity location (*j*) are located within network arcs while six flexible activity locations are located at the network nodes. Table 1 provides the attractiveness levels of the flexible activity locations and the input parameter values. (Figure 2 provides the flexible activity location labels to avoid additional clutter in Figure 1.)

Table 2 provides results from calculating an individual's accessibility benefits (case 1) and results from interpolating benefits within an arc (case 2). The former assumes a space-time prism dictated by travel from the first fixed activity to the second fixed activity with a time budget of 30 minutes. The latter interpolates benefit levels within an arc corresponding to varying locations for the second fixed activity location (e.g., varying workplace locations for a home-based travel episode). The time budget for this case is 60 minutes. The interpolation uses a fixed interval of ρ=0.2, generating six interpolated values for the arc. Figure 2 provides the locations corresponding to the interpolated values listed in Table 2. All three AMs exhibit decreasing accessibility benefit as the second fixed activity (e.g., workplace) becomes more distant from the first fixed activity (e.g., home).

Figure 3 and Figure 4 provide examples of an attribute-based location query (case 3). In these examples, the second fixed activity is held constant, implying that the highlighted network arcs correspond to varying locations of the first fixed activity (e.g., varying home locations for individuals working at the second fixed activity site). The time budget for this example is 30 minutes. Figure 3 shows all locations within the network with an AM_2 value greater than or equal to 15 while Figure 4 shows all locations within the network with an AM_3 value greater than or equal to 35. Both calculations use an error tolerance of 0.01 in the bisection search. Note from Figure 3 that high benefit levels correspond to location proximal to flexible activity locations with high attractiveness. Figure 4 exhibits a similar pattern, although the more generous benefit level specified (for this AM) generates a region encompassing the second fixed activity location as well as a flexible activity location with relatively low attractiveness.

6. DISCUSSION AND CONCLUSION

The AMs developed in this paper are rigorous, realistic and easily computed techniques for measuring individual accessibility. With respect to rigor, the AMs are consistent with the Weibull axiomatic system for accessibility measurement. Although the Weibull system is limited with respect to admissible behaviors, it ensures internal and external consistency within its behavioral scope. These are valuable properties given the disparate and often inconsistent ways in which accessibility has been defined and measured in the literature. Also, the AMs provide explicit measures of the benefits derived from accessibility. AM_1 is consistent with random utility measures of expected maximum utility and with consumer surplus (although not strict willingness-to-pay) measures. AM_2 and AM_3 are consistent with Wilson's (1976) measure of locational benefits derived from spatial interaction. This framework supports interpreting the AMs as ratio measures of accessibility. Therefore, we can use the measures to determine accessibility differences beyond simple ordinal relationships.

With respect to realism, the AMs incorporate the Hägerstrand space-time prism approach to delimiting constraints on individual’s activity participation. This provides a substantial increase in realism over traditional AMs that assume implicitly that all alternatives are available to individuals. Calculating the measures within a network enhances this realism since the transportation network dictates travel paths, velocities and activity locations in an urban environment. As noted previously, we can take into account travel times conditioned by current or projected network flows and congestion.

The previous section of this paper demonstrated that these network-based calculations are computationally tractable.

Another factor that enhances the applicability of the AMs in transportation planning and analysis is the ease of parameter estimation. Well-established parameter estimation techniques available in both the multinomial logit and spatial interaction modeling frameworks. Ben-Akiva and Lerman (1985) and Fotheringham and O'Kelly (1989) provide discussions of parameter estimation within both domains, respectively.

A potential application difficulty is measuring individuals' time budgets. As Thill and Horowitz (1997a) point out, these data are not available from standard revealed-preference data. Consequently, they treat temporal constraints as unobservable (at least directly) within a random utility framework. By assuming a probability distribution for these unobservable time budgets, their destination choice model allows inference of temporal choice constraints based on revealed travel times to alternatives.

Although standard revealed preference data does not allow direct measurement of time budgets, it seems feasible to derive these data from other data collection methods. Recall that the AMs can accommodate two time budget measurements, namely based on mandatory end times and start times $\left(t_i \text{and}\, t_j\right)$ for fixed activities or based on an aggregate reported time budget (T') for the travel episode. These former data can be collected through travel diary methods. For example, we can ask individuals to note whether a recorded activity was mandatory or flexible. We can also ask whether the start and end times were mandatory; if not, we can ask for these times. Travel diary data collection is becoming popular due to resurgence in activity-based analysis (Greaves and Stopher 1998). Aggregate time budgets (T') can also be derived from travel diary data collection methods or by asking individuals about time availability (as well as the start and end locations) when a trip occurred within standard revealed-preference survey instruments. Although it may be argued that these self-reported data are fraught with error, it is an open research question whether these errors are greater than those incurred by assuming an unobservable time budget subject to a stochastic disturbance.

As noted in the introduction, this paper focuses on individual-level accessibility measures. Despite their individual-level nature, it is still possible to use the measures for large-scale transportation planning and analysis. It is in this respect that the true power of a GIS-based implementation will be realized. A GIS can be used to summarize the individual-level AMs derived within the network and tie these measures to other, socio-economic data. In particular, the visualization capabilities of the GIS can be exploited to derive powerful cartographic summaries of accessibility variations across the network. The individual-level measures can also be tied to address-indexed data files or to the census geography through the georeferencing capabilities of the GIS. We are directing our current research efforts to these visualization and data integration questions.

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Table 1: Input data for AM example

Flexible activity location[1]	Attractiveness	Utility function parameters
1	7000	Alpha: 0.5 Beta: 0.7 Lambda: 0.9
2	5000	
3	3000	
4	5000	
5	10,000	

[1] Figure 2 provides the flexible activity labels

Table 2: Case 1 (individual benefit) and case 2 (benefit interpolation) results

Case 1 Time budget = 30 minutes	AM_1	AM_2	AM_3
	14.685	20.557	13.216
Case 2			
Time budget = 60 minutes, $\rho = 0.2$	AM_1	AM_2	AM_3
Location 1	19.342	35.101	17.388
Location 2	15.859	28.998	14.234
Location 3	13.024	24.060	11.651
Location 4	10.721	20.090	9.538
Location 5	8.855	16.928	7.807
Location 6	7.352	14.446	6.391

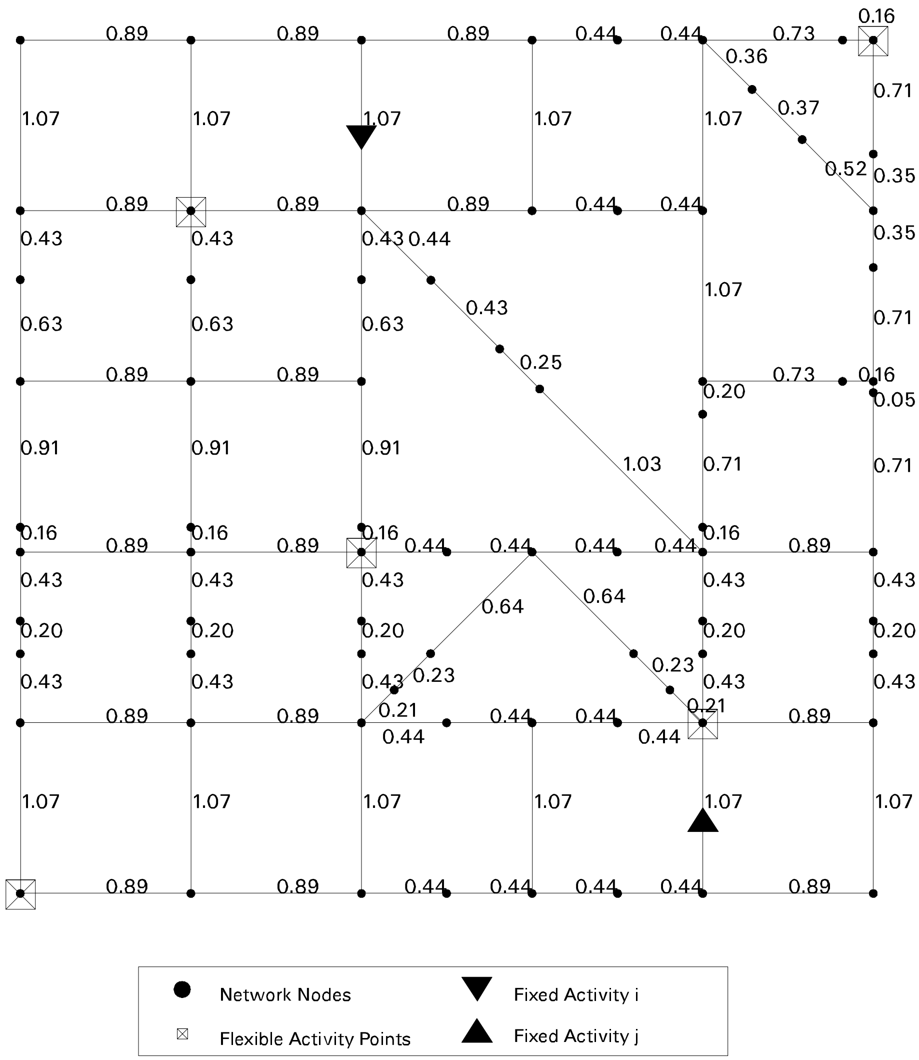

Figure 1: Example network for AM calculations

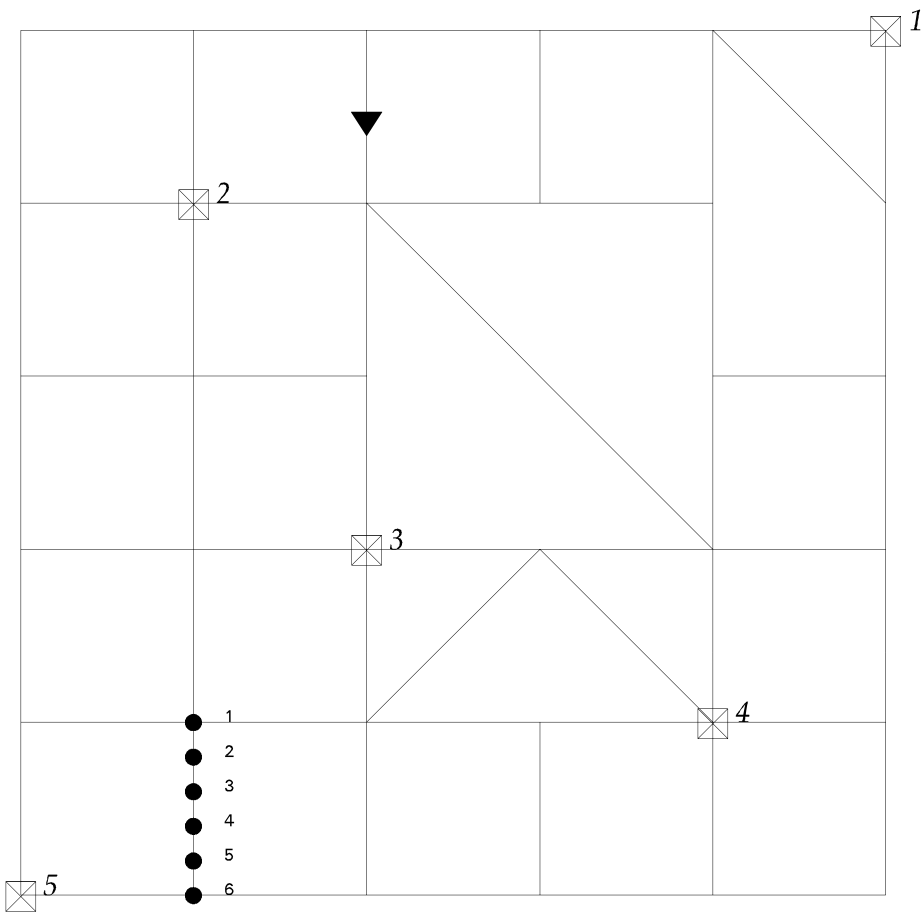

Figure 2: Interpolated locations within network

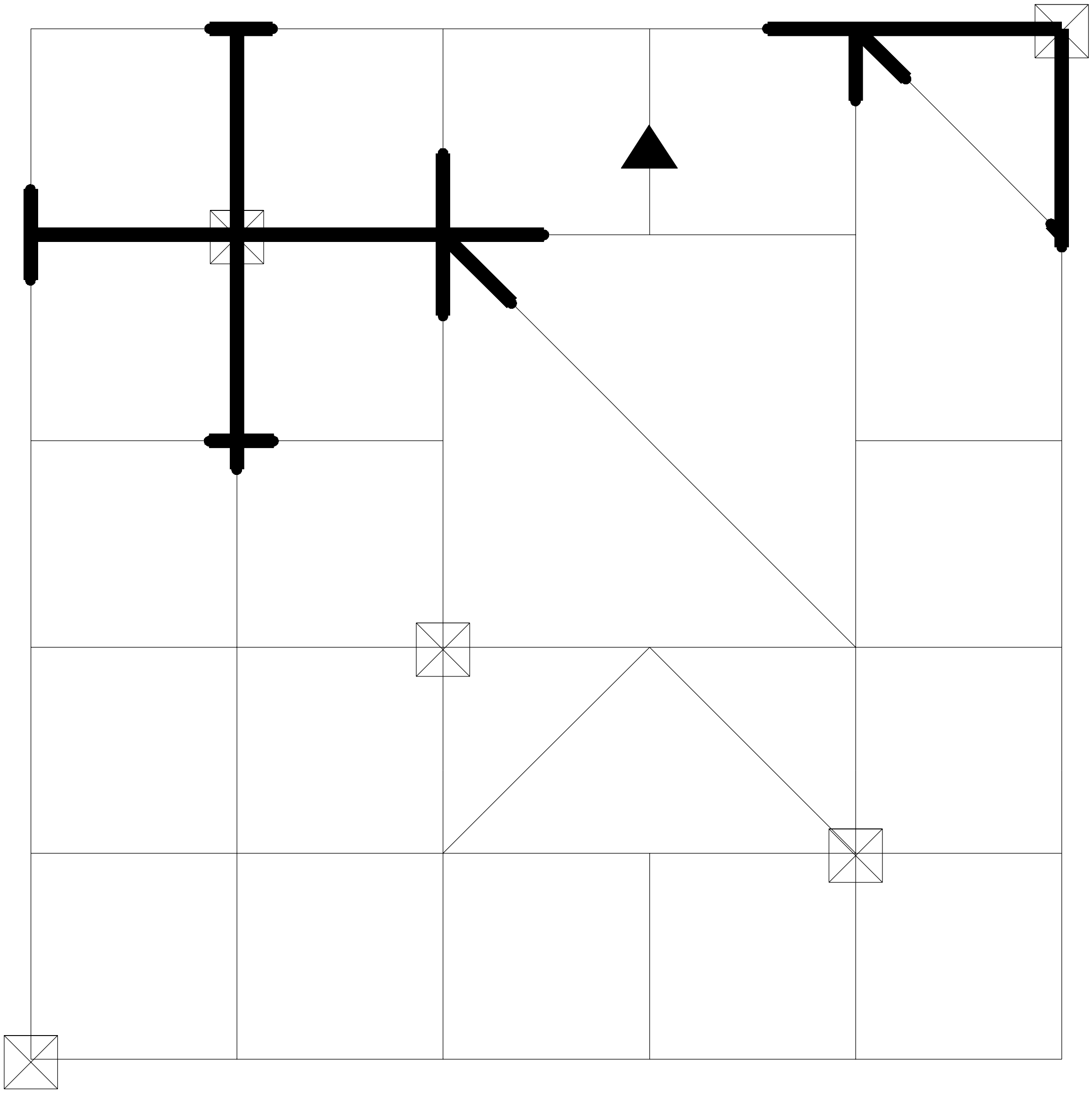

Figure 3: Example attribute-based query: AM_2 greater than or equal to 15

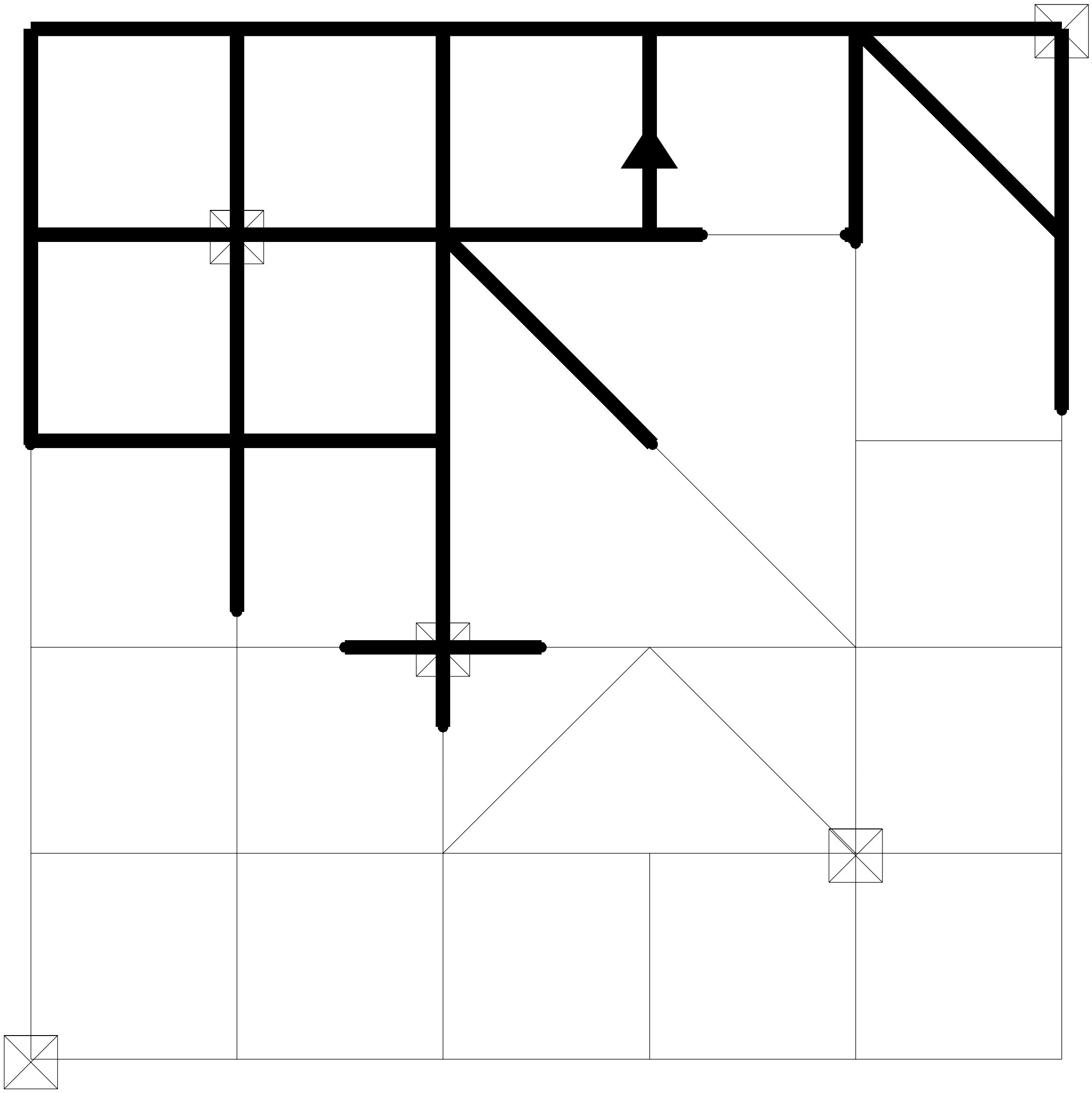

Figure 4: Example attribute-based query: AM_3 greater than or equal to 35

FOOTNOTES

[1] Note that we could take the natural log of equation (26) to obtain the typical linear-in-parameters form (similar to equation (27)) required for some estimation packages. However, the nonlinear form is more consistent with the basic theory and does not require additional conditions (see equation (27)) that are more difficult to handle within the log-sum format.